STRESS DISTRIBUTION AT THE TIP OF A GROWING CRACK

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ABSTRACT: High-speed motion-picture photography and an optical method of stress analysis have been used to study the distribution of elastic stress fields at the tip of a crack growing at a fast rate. The existence of several specific properties of the field characteristic of fast crack propagation rates has been established, and the results obtained are used to explain the branching of cracks.

One of the most important directions of research leading to the crystallization of concepts of the nature of fracture is the study of the final loading stage, i.e., the propagation of the fracture crack. Studies of the kinetics of crack propagation present considerable difficulties, associated mainly with the very fast crack propagation rates. As a result, the number of investigations in this field is disproportionately small in relation to studies of problems of strength in general and to the volume of experimental data required to substantiate the highly developed theory of this problem.

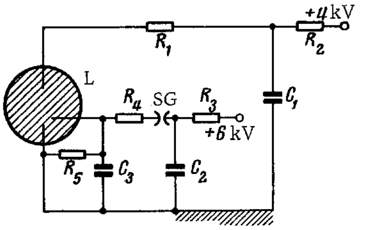

Fig. 1

The main parameters of crack propagation, i.e., its rate and acceleration, fracture energy, crack propagation path, etc., are determined by the stress distribution at the crack tip. These parameters in the case of a static crack have often been considered in various theoretical and experimental studies [1-3], whose general conclusion is that the stress intensity in the vicinity of a crack tip is inversely proportional to the square root of the distance of a given point from the tip, and that the maximum stresses are oriented at 45°-60° to the crack axis.

As far as the problem of fracture is concerned, the stress distribution surrounding a running crack is the most important factor. Studies of this kind were previously carried out on metals and polymers. Of a considerable interest in this connection are the results obtained by Wells and Pose [4] who reached a conclusion that stress fields surrounding static and polymers. Of a considerable interest in this connection are the results obtained by Wells and Pose [4] who reached a conclusion that stress fields surrounding static and dynamic cracks are identical. This conclusion obviously contradicts the results of Ioffe's theoretical work which demand a change in the elastic field at higher crack propagation rates, and the hypotheses of Graggs [6] and Broberg[7], according to which a change to an accelerated mode of fracture propagation may occur at a constant stress if a sufficiently fast propagation rate is reached. One should mention also the work of Akita and Ikeda [8] who observed some differences between stress fields of "fast" and "slow" cracks.

In our opinion, the mode of crack propagation should have a fundamental effect on the structure of elastic stress fields at the crack tip. This should be manifested both at speeds much below the branching threshold and at the moment when branching takes place. The latter is quite a feasible proposition, if only because the very process of branching is bound to be a result of a definite rearrangement of the elastic stress field at the crack tip. Although the branching process has been studied by several workers [9-11], we know of no investigations of the stress field at the tip of a branching crack. It was shown theoretically by Barenblatt that even before the onset of branching one should expect the appearance of stress field anomalies in the form of an extended field located in front of the crack, in the direction of its axis.

The aim of the present investigation was to study the stress field near a fast propagating crack and to determine the causes of branching.

Experimental technique. The characteristics of stress fields at the tip of a fast crack were studied by an optical method of stress analysis, 3-mm thick celluloid sheet being used as the photoactive material. The shear fringe scale value of the model was σ_0 = 100 kg/cm^2.

Celluloid sheet specimens 50 mm wide were tested to rupture on a tensile testing machine, under stresses of about 500 kg/cm^2. To increase the proportionality limit of the material, the tests were carried out at temperatures between 0 and −5° C.

The stresses were determined by a birefringence method [12] using a circular polariscope. The isochromatics were recorded by high-speed photography and motion-picture photography. Unfortunately, this method does not enable one separately to determine the magnitude of the principal stresses σ_1 and σ_2, and gives only the maximum tangential stress $\tau_{\max} = (\sigma_1 - \sigma_2)/2$.

Two series of tests were carried out. In one of them, motion pictures of the stress field at the tip of a crack at the moment of its propagation were obtained; this was done with the aid of flash illumination (a ISSh-100-3 flash lamp producing a flash lasting $4 \cdot 10^{-6}$ sec with a flash energy of 0.8 joule).

The flash unit was ignited with the aid of a special device (Fig. 1) which operated in the following way: the knife edges of a spark gap SG were attached to the specimen and connected in series to a semispherical capacitor, at which the crack originated; at the moment at which the crack ran under these knife edges, a disruptive discharge

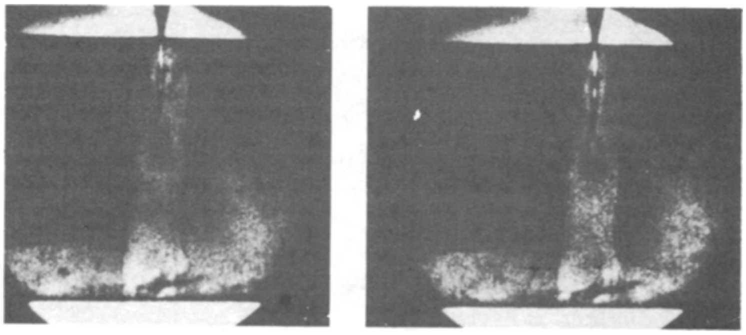

Fig. 2

Fig. 3

took place through the gap formed by the gap; the resulting high-voltage pulse was transmitted through a delay element R_4C_3 to the lamp L and ignited it; the ignition time lag could be adjusted by varying the resistance R_4; the isochromatic pattern was photographed at the moment of ignition.

In the second series of tests, motion pictures of the process of fracture of celluloid specimens were taken in polarized light with a type SFP-2m camera at speeds of 120 and 60 000 frames/sec. The illumination was provided by 20 IFK-120 lamps, grouped to form a special battery which produced a uniform light beam. The flash energy in this case was 1800 joules.

The type SFP-2m camera does not operate in the "driven" mode, so that the fracture must be triggered off by the camera itself. When a stress of about 500 kg/cm^2 (YP of the material) is reached, a pulse produced by the camera ignites a detonator which strikes the specimen edge with a 200-mm long knife blade, thus initiating a crack at the point of impact. The ignition of the light source was synchronized with the onset of fracture either by direct switching-on with a contact device attached to the knife blade or by the initial pulse produced by the camera and electronically delayed for the period of $200 \cdot 10^{-6}$ sec required for the detonator to initiate fracture.

The SFP-2m lens holder was slightly modified so that it was possible to take motion pictures at a distance of 70 cm and still obtain images of satisfactory quality.

Test results and discussion. 1. It is known that brittle fracture is always accompanied by plastic deformation in the region adjacent to a crack. This fact must be taken into account in considering any fracture processes. From this point of view, the material studied in the present investigation closely resembles metals in that the crack propagation in it is associated with the existence of a plastic deformation zone, clearly visible on a number of photographs (Fig. 2): The black opaque border surrounding a crack represents deformed celluloid. The length of the plastic strain zone in front of the crack may, according to our calculations, reach 8 mm. According to [13], the effective size of a crack in the case of quasi-brittle fracture is larger than its actual size by the size of the plastically deformed zone whose boundary should be regarded as the crack surface. The material beyond the confines of the effective crack is in the elastically-stressed state. This explains the fact that circular isochromatics determining the elastic field of a specimen originate, in our case, at the front of the plastic zone. Rings of this kind, emerging in front and at the sides of the crack axis were extensively studied, for instance, in [4], and will not be discussed here. We were mainly concerned with the stress distribution on the path of a crack in front of its tip, since the development of the fracture process, its rate and the character of the crack (single or branching) depend on the behavior of the material in this particular region.

Using a flash photography method, it was possible to obtain photoelastic stress patterns at the tip of a growing crack, which are reproduced in Fig. 3a,b.

One's attention is drawn to the narrow light band (up to 15 mm long and about 2 mm wide) in front of the crack. Three different explanations of the nature of this band can be postulated: 1) the band is formed as a result of plastic deformation of the material; 2) the band is a region of "silver" cracks [14]; 3) the band is a low-order isochromatic. The first of these hypotheses must be rejected, since—as was shown earlier—the plastic strain zone is opaque on the photographs; we have often observed that any degree of plastic deformation makes celluloid less transparent. As regards the second hypothesis, it should be pointed out that "silver" crack photographed with transmitted light also ought to appear black (e.g., see [14]).

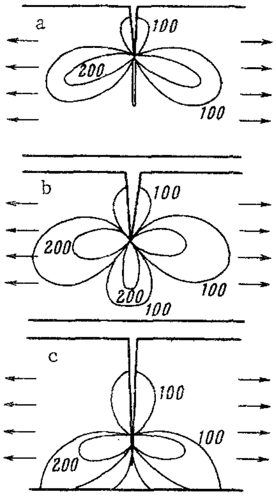

Fig. 4

Consequently, the band under consideration represents an isochromatic—a stress "wedge" (Fig. 4a), in which the tangential stress level (determined by the fringe method) reaches 150 kg/mm^2. An elastic stress field of this form is characteristic only for fracture at very fast rates. Motion pictures taken at a speed of 60 000 frames/sec showed that the "wedge" appears at cracks propagation rates of about 300 m/sec. Figure 3b shows a photoelastic stress pattern obtained under the same conditions for a static crack. As rightly pointed out in [4], the stress distribution in the static and dynamic cases is very similar, but the pattern for a static crack does not show a "wedgelike" isochromatic.[*]

It can be postulated that this anomaly of the elastic stress field near a rapidly growing crack plays an important role in the process of fracture: In a narrow zone formed by the "wedge" there is a concentration of strain energy and the yield point is reached earlier, which facilitates the formation of microcracks.

2. Somewhat different was the character of stress fields surrounding cracks which grow at rates approaching the highest possible. (The crack propagation rate was increased by reducing the test temperature to $-5°$ C.) Using high-speed (120 000 frames/sec) motion-picture photography, we were able to determine the stress distribution in the case of a crack growing at a rate of 500 m/sec. Motion pictures of this process reproduced in Fig. 5 show that no formation of the "wedge" discussed above takes place at such fast crack propagation rates. In

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[*] A "wedge" similar to that described above can be seen on photographs reproduced in [4, 15].

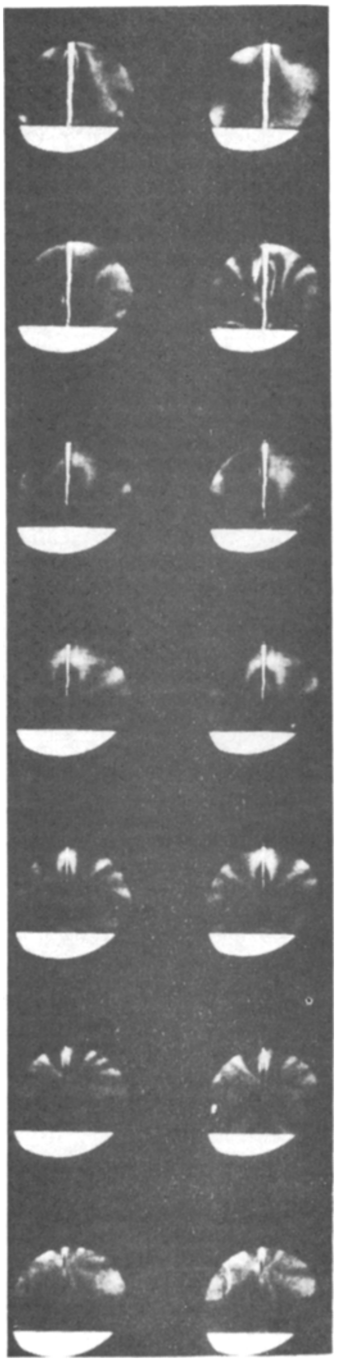

Fig. 5

this case, in front of the effective crack tip there appears a zone of circular isochromatics which moves with the tip. The external boundaries of this zone are at 45° to the crack axis; its linear dimensions reach 30 mm. The disposition of ischromatics at the tip of a fast growing crack is shown in Fig. 4c. It was found possible to determine the stress level in the circular isochromatics zone: The stress linearly decreases with increasing distance from the crack tip (Fig. 6).

A similar distribution of isochromatics in front of a crack tip was theoretically determined by Becker [16]. According to his calculations, such a distribution is characteristic of crack propagation at rates approaching the fastest possible (i.e., the velocity of Rayleigh waves in a given material). In our case, the formation of circular isochromatics takes place at a crack propagation rate equal to 0.9 of the Rayleigh velocity for celluloid.

It is significant that in all cases, when circular isochromatics were observed, intense branching of the cracks took place; it started usually after the crack had traversed one fifth of the specimen width. Typical appearance of a fractured celluloid specimen is shown in Fig. 7.

The characteristics of an elastic stress field around a fast running crack can be related to the branching effect. The circular isochromatics delineate in front of the fracture zone a region in which the stress distribution is the same both in the direction of the crack axis and at 45° to it. Such a distribution makes equally probable rectilinear growth of a crack and branching at 45° C to the initial direction. Theoretical considerations led Ioffe [5] to the same conclusion, but the existence of a region of equal stresses was not previously demonstrated by experiment.

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Consequently, the cause of the branching of a fast growing crack is inherent in the very character of its propagation, i.e., in the character of the elastic stress field surrounding the fracture zone.

According to a different theory [9], the branching of a crack is a result of an interaction between the stress field at the crack tip and elastic pulses reflected by the specimen edges. Undoubtedly, this interaction may bend the crack trajectory and lead to its branching; it cannot, however, explain all the observed effects. Figure 5 shows that the influence of the specimen edge on the elastic stress field is manifested only when the distance between this edge and the crack tip is small (about 1 cm); see also Fig. 4c. This influence is manifested in the following way: When the outer circular isochromatic reaches the specimen edge the inner isochromatic contracts and becomes transformed into a "wedge" described above. A crack begins to branch long before this distortion takes place.

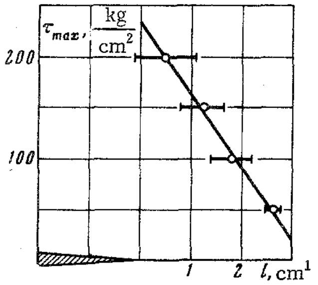

Fig. 6

Moreover, if reflected elastic pulses were the only cause of the branching, the location of the first fork should depend on the specimen width. No evidence of such dependence was observed in a series of control tensile tests carried out on polymethacrylate specimens of various widths; (this polymer was used because cracks in it branch readily).

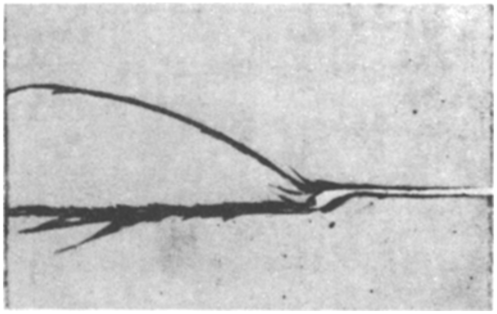

Fig. 7

The above considerations lead to the following conclusions:

1. A stress field near a fast growing crack is substantially different from a crack surrounding a static crack; a "wedge," i.e., a narrow band of increased tangential stresses appears in front of a "fast" crack.
2. At faster crack propagation rates, a zone of circular isochromatics is formed in front of a growing crack. The stress distribution in this zone is the same in different directions relative to the crack axis, and the stress level linearly decreases with the distance from the crack tip.
3. Branching of a crack takes place at the moment of the formation of a zone of equal stresses. The cause of branching lies in the very character of crack propagation, i.e., in the rearrangement of the stress field at high propagation rates.

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